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Analysis of an Array of Four Microstrip Patch Resonators Printed on an Anisotropic Substrate

Yinchao Chen and Benjamin Beker

***Abstract*—The spectral-domain approach is applied to the analysis of a four microstrip patch resonator array that is printed on anisotropic substrate. Basis functions are carefully chosen to accurately represent the current distribution on each patch, corresponding to every symmetry of the structure. Ample numerical results are presented which show the effects of geometrical and anisotropic medium parameters on dominant resonant frequencies of the microstrip patch array.**

I. INTRODUCTION

Electromagnetic properties of anisotropic materials have been recently investigated in many important applications, including microwave and millimeter wave integrated circuits [1], microstrip antennas [2], and microstrip resonators [3]. To efficiently formulate boundary-value problems in wave propagation involving both isotropic and anisotropic media, several variations of matrix representation to Maxwell's equations have been proposed [4]–[7].

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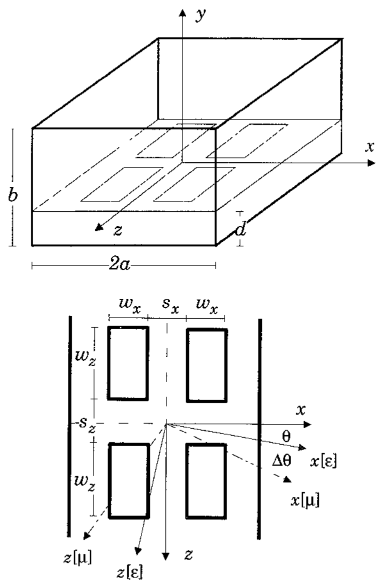

Fig. 1. Geometry of a four microstrip patch resonator array.

In this paper, the spectral domain approach and matrix operators are employed to analyze four patch resonator arrays that are printed on anisotropic substrates. Microstrip patch resonators find important applications in MICs as resonant elements of oscillator and filter circuits. Anisotropic nature of the substrate allows an added degree of freedom in the design of the array. It allows for electronic rather mechanical tuning of the resonator by changing the applied biasing dc field. The existing literature dealing with this subject is rather limited [3], thereby providing motivation for this work.

The resonator problem is formulated in the spectral domain in terms of two components of the E-field that are tangential to the substrate interface. The remaining field components are obtained from Maxwell's curl equations. To calculate resonant frequencies efficiently, basis functions for the currents on all four patches are systematically constructed to represent even-even, even-odd, odd-even, and odd-odd modes. Dominant resonant frequencies are computed for different geometrical and substrate material parameters, showing their effect on each mode.

II. FORMULATION OF THE RESONATOR PROBLEM

Consider four rectangular patches with dimensions, (w_x, w_z), and separated by (s_x, s_z), that are printed on an anisotropic substrate enclosed within a rectangular waveguide shown in Fig. 1. The substrate is characterized by $[\varepsilon]$ and $[\mu]$ tensors whose xy, yx, yz, and zy elements are zero, but it includes missaligned with the x and z axes of the resonator (see Fig. 1) [8].

To formulate the boundary-value problem in the spectral domain, the following 2D Fourier transform is employed:

$$\tilde{\xi}(\alpha_n, y, \beta) := \int_{-\infty}^{-\infty} \int_{-a}^{a} \xi(x, y, z) e^{\jmath\alpha_n x + \jmath\beta z} \, dx\, dz. \tag{1}$$

Unlike [9] where isotropic substrates are considered, herein, summation index n takes on all integer values from minus to plus infinity due

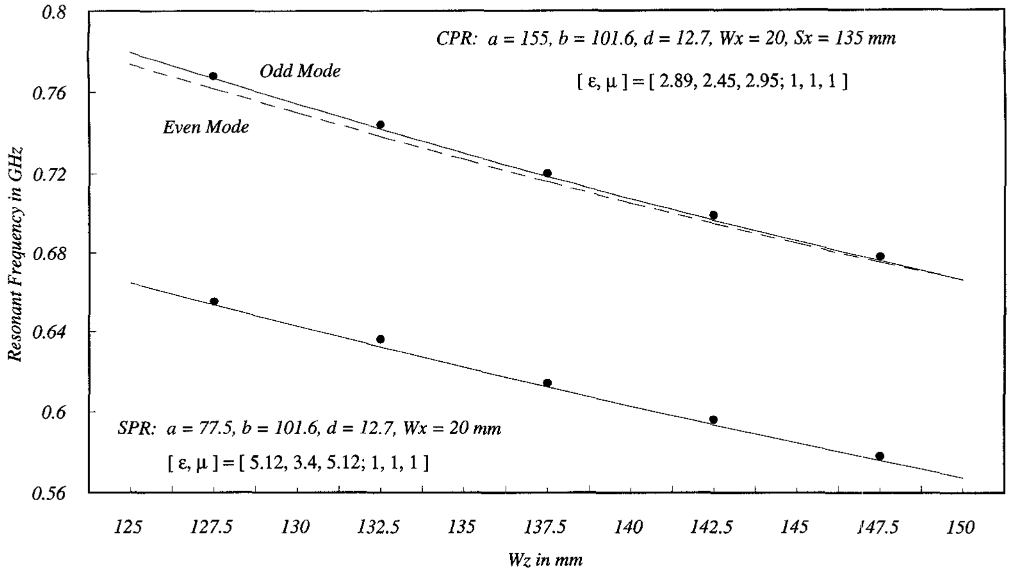

Fig. 2. f_r of single (SPR) and coupled (CPR) microstrip patch resonators as functions of w_z (solid lines correspond to present method, circles denote data from [3]).

to the asymmetry of the Green's function introduced by off-diagonal elements of the material tensors.

Once transformed to the spectral domain, the space domain vector wave equation [8], (5a) with aforementioned tensor elements set to zero leads directly to two coupled equations for $\tilde{E}_x$ and $\tilde{E}_z$. Solution for $\tilde{E}_x$ and $\tilde{E}_z$ determines the field in the planar anisotropic region, which can then be used to find remaining field components and to enforce the boundary conditions at the interface to derive the impedance Green's function:

$$\begin{bmatrix} \tilde{Z}_{zz}(\alpha_n,\beta) & \tilde{Z}_{zx}(\alpha_n,\beta) \\ \tilde{Z}_{xz}(\alpha_n,\beta) & \tilde{Z}_{xx}(\alpha_n,\beta) \end{bmatrix} \begin{bmatrix} \tilde{J}_z(\alpha_n,\beta) \\ \tilde{J}_x(\alpha_n,\beta) \end{bmatrix} = \begin{bmatrix} \tilde{E}_z(\alpha_n,\beta) \\ \tilde{E}_x(\alpha_n,\beta) \end{bmatrix}, \quad (2)$$

whose elements are similar to those in [8].

Finally, Galerkin's method is implemented to set up a system of linear equations whose determinant contains the resonant frequencies. To this end, the unknown currents, $(\tilde{J}_x, \tilde{J}_z)$, are expanded in terms of basis functions with unknown coefficients (b_m, a_m):

$$\tilde{J}_{(z,x)}(\alpha_n,\beta) = \sum_{m=1}^{(M,N)} (a_m, b_m)\tilde{J}_{(zm,xm)}(\alpha_n,\beta). \quad \text{(3a, b)}$$

These basis functions are defined over each patch so that they satisfy required edge conditions. Subsequent inner products and use of Paserval's theorem lead to two sets of homogenous linear equations:

$$\sum_{m=1}^{M} \begin{pmatrix} K_{im}^{zz} \\ K_{im}^{xz} \end{pmatrix} a_m + \sum_{m=1}^{N} \begin{pmatrix} K_{im}^{zx} \\ K_{im}^{xx} \end{pmatrix} b_m = 0 \quad \begin{pmatrix} i = 1,2,\ldots,M \\ i = 1,2,\ldots,N \end{pmatrix} \quad (4)$$

with

$$\begin{pmatrix} K_{im}^{zz}(f_r) \\ K_{im}^{zx}(f_r) \end{pmatrix} = \sum_{m=-\infty}^{+\infty} \int_{-\infty}^{+\infty} \tilde{J}_{zi}(\alpha_n,\beta) \begin{pmatrix} \tilde{Z}_{zz}(\alpha_n,\beta) \\ \tilde{Z}_{zx}(\alpha_n,\beta) \end{pmatrix} \times \begin{pmatrix} \tilde{J}_{zm}(\alpha_n,\beta) \\ \tilde{J}_{xm}(\alpha_n,\beta) \end{pmatrix} d\beta \quad \text{(5a, b)}$$

$$\begin{pmatrix} K_{im}^{xz}(f_r) \\ K_{im}^{xx}(f_r) \end{pmatrix} = \sum_{m=-\infty}^{+\infty} \int_{-\infty}^{+\infty} \tilde{J}_{xi}(\alpha_n,\beta) \begin{pmatrix} \tilde{Z}_{xz}(\alpha_n,\beta) \\ \tilde{Z}_{xx}(\alpha_n,\beta) \end{pmatrix} \times \begin{pmatrix} \tilde{J}_{zm}(\alpha_n,\beta) \\ \tilde{J}_{xm}(\alpha_n,\beta) \end{pmatrix} d\beta, \quad \text{(6c, d)}$$

where f_r is the resonant frequency of the structure. Matrix (4) can now be solved by setting its determinant to zero and searching for the root to find the resonant frequency.

III. Basis Functions

Proper choice of basis functions is a key factor for efficiently calculating accurate resonant frequencies, when using the spectral domain technique. A set of good basis functions for a single patch that satisfy the edge conditions and have fast convergence properties were proposed in [9], [10].

In this paper, basis functions similar to those that were defined in [10] for a single patch will be employed. The symmetry of the structure allows four different modes to exist, which mathematically are obtained by placing either electric or magnetic walls in the $x = 0$ and $z = 0$ planes. Consequently, the basis functions for the four patch array can be written as

$$J_z(x,z) = \left\{ \frac{U_{x1}}{\sqrt{1 - \left[\frac{2(x+b_x)}{w_x}\right]^2}} \begin{pmatrix} + \\ + \\ - \\ - \end{pmatrix} \frac{U_{x2}}{\sqrt{1 - \left[\frac{2(x-b_x)}{w_x}\right]^2}} \right\} \times \left\{ \frac{\cos\left[\frac{\pi}{w_z}(z+b_z)\right]U_{z1}}{\sqrt{1 - \left[\frac{2(z+b_z)}{w_z}\right]^2}} \begin{pmatrix} - \\ + \\ - \\ + \end{pmatrix} \times \frac{\cos\left[\frac{\pi}{w_z}(z-b_z)\right]U_{z2}}{\sqrt{1 - \left[\frac{2(z-b_z)}{w_z}\right]^2}} \right\} \times \begin{pmatrix} ee \\ eo \\ oe \\ oo \end{pmatrix} \quad \text{(7a)}$$

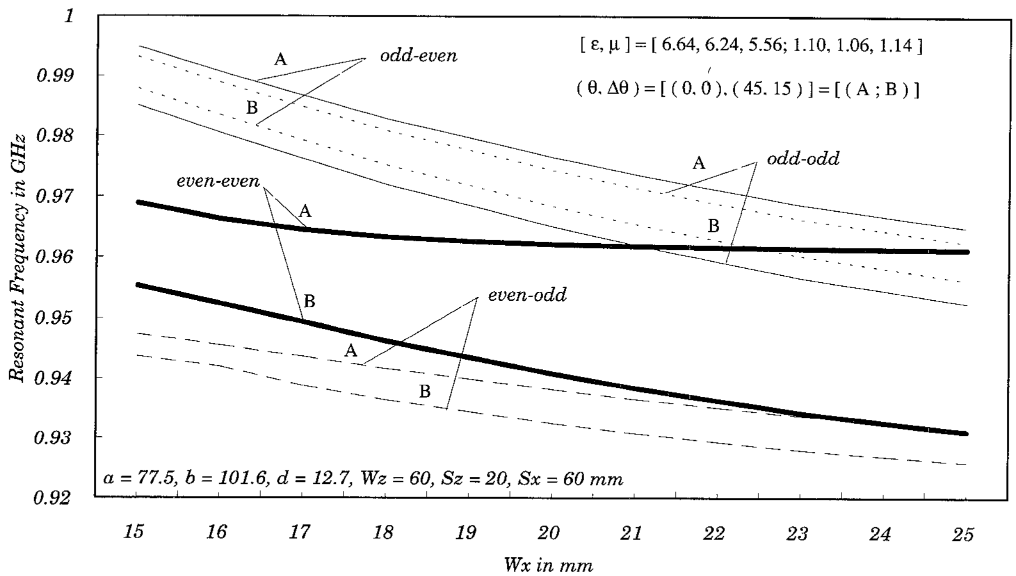

Fig. 3. f_r of all (even-even, even-odd, odd-even, and odd-odd) modes of a patch resonator as a function of w_x with and without misalignment.

$$J_x(x,z) = \left\{ \frac{\sin\left[\frac{2\pi}{w_x}(x+b_x)\right]U_{x1}}{\sqrt{1-\left[\frac{2(x+b_x)}{w_x}\right]^2}} \begin{pmatrix} - \\ - \\ + \\ + \end{pmatrix} \times \frac{\sin\left[\frac{2\pi}{w_x}(x-b_x)\right]U_{x2}}{\sqrt{1-\left[\frac{2(x-b_x)}{w_x}\right]^2}} \right\} \times \left\{ \frac{\sin\left[\frac{\pi}{w_z}(z+b_z)\right]U_{z1}}{\sqrt{1-\left[\frac{2(z+b_z)}{w_z}\right]^2}} \begin{pmatrix} + \\ - \\ + \\ - \end{pmatrix} \times \frac{\sin\left[\frac{\pi}{w_z}(z-b_z)\right]U_{z2}}{\sqrt{1-\left[\frac{2(z-b_z)}{w_z}\right]^2}} \right\} \times \begin{pmatrix} ee \\ eo \\ oe \\ oo \end{pmatrix} \tag{7b}$$

where ee, eo, oe, and oo correspond to different modes of the structure, i.e., to even-even, even-odd, odd-even, and odd-odd modes. Mode designation refers to symmetries about the $x = 0$ and $z = 0$ planes, respectively. All geometrical parameters are defined in Fig. 1, with $b_{x,z} = 0.5(s_{x,z} + w_{x,z})$, and where the Heaviside unit step functions are given by

$$U_{x(1,2)} = \pm\left\{U\left(x \pm w_x \pm \frac{s_x}{2}\right) - U\left(x \pm \frac{s_x}{2}\right)\right\} \quad \text{(8a, b)}$$
$$U_{z(1,2)} = \pm\left\{U\left(z \pm w_z \pm \frac{s_z}{2}\right) - U\left(z \pm \frac{s_z}{2}\right)\right\}. \quad \text{(8a, b)}$$

Careful examination of the current distribution reveals that all four patches will have same functional forms, except for origin translation. The signs of current basis functions for all symmetries on an individual patch are summarized below:

The entries in the table correspond to signs in (7a), (7b) for each mode, while their pattern within the table corresponds to the geometrical location of each patch in an array shown in Fig. 1.

TABLE I
SIGNS OF CURRENT $J_{Z,X}$ ON THE FOUR PATCHES FOR DIFFERENT MODES

Even-Even		Even-Odd		Odd-Even		Odd-Odd	
(+,+)	(+,-)	(+,+)	(+,-)	(+,+)	(-,+)	(+,+)	(-,+)
(-,+)	(-,-)	(+,-)	(+,+)	(-,+)	(+,+)	(+,-)	(-,-)

IV. NUMERICAL RESULTS

To validate the formulation and its spectral domain implementation, two different resonators are considered. They include a single patch resonator (SPR) mode and the odd mode of a coupled patch resonator (CPR) with resonant frequency data compared to those of reference [3]. Figure 2 displays results of the validation study, showing a good agreement for both structures. Note that the width of the coupled (two patch) resonator was doubled for comparison purposes.

In all subsequent examples, waveguide dimensions ($2a$ and b) are held constant ($a = 77.5$, $b = 101.6$, $d = 12.7$ mm), as are the principal elements of permittivity and permeability tensors of the substrate $[\varepsilon] = [6.64, 6.24, 5.56]$ and $[\mu] = [1.1, 1.06, 1.14]$. The only variables are patch dimensions, separation between them, and the misalignment between the principal axes of the substrate and the guide in the xz plane (see Fig. 1).

First, effects of patch dimension w_x on the resonant frequencies of the array are investigated, with results displayed in Fig. 3. As w_x changes, the resonant frequency curves of the even-even and even-odd modes (for misalignment angle A) remain much flatter than all other curves, allowing for frequency crossover to occur.

Finally, effects of patch separation were examined and are summarized in Fig. 4. As expected, when s_z is small, resonant frequencies of all modes have significantly different and rapidly varying values, which indicates strong coupling between the currents on neighboring patches. However, as the patch separation along z, (s_z), becomes larger, the coupling weakens, and the resonant frequencies of the patch array reach constant values.

In contrast to the changes in s_z, effects of varying patch separation in the x-direction are more profound due to the presence of the sidewalls. Figure 4 clearly illustrates what happens when s_x increases. Notice that for small values of s_x resonant frequencies of odd-even and odd-odd modes are considerably different from those of the even-

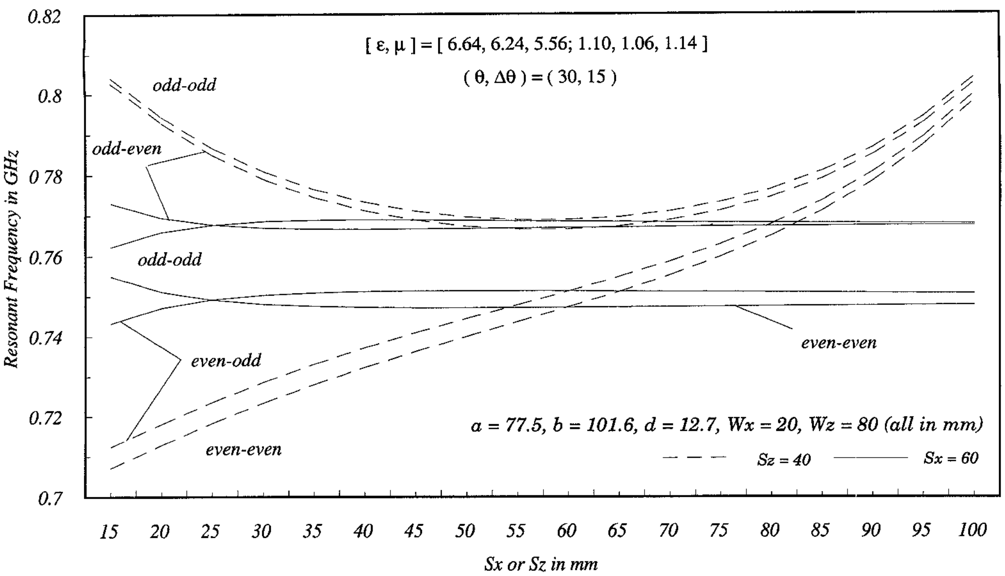

Fig. 4. f_r of all modes of a patch resonator as a function of s_z (solid lines) and s_x (broken lines).

even and even-odd modes. Also in this region, they exhibit totally contrasting behavior—namely, f_rs of *o-e* and *o-o* modes decrease, while those of *e-e* and *e-o* steadily increase. As the transverse separation between the patches increases, resonant frequencies of the array are significantly influenced by the sidewalls of the waveguide. They all tend to increase towards the same value.

V. Conclusion

Resonant properties of a four microstrip patch resonator array, printed on an anisotropic substrate, were analyzed. Good agreement with limited published data was found, and ample numerical results for resonant frequencies of the resonator array were presented as functions on the geometrical and substrate material parameters.

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A Technique for Minimizing Intermodulation Distortion of GaAs FET's

Haruhiko Koizumi, Shunsuke Nagata and Kunihiko Kanazawa

***Abstract*—This paper describes the theory to minimize the intermodulation distortion under certain current bias condition for GaAs FET's. A device-parametric study has been done to obtain the general equation that provides the lowest distortion condition as a function of an operating current. Based on the present theory, FET parameters have been designed practically.**

I. Introduction

In recent years, the microwave GaAs MESFET has been widely used for a variety of mobile communication systems. In those portable systems, what is most important is to reduce the operating current since the battery running time is a major concern. In addition, recent digital communication systems require the minimization of an intermodulation (IM) distortion in order to improve the error rate. However, it is difficult that GaAs FET's provide the low current and the low distortion characteristic at the same time. Therefore, the goal

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